

Title: Rational Equations, Level II.

Class: Math 107 or Math 120 or Math 137

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Instructions to Tutor: Read instructions and follow all steps for each problem exactly as given.

Keywords/Tags: solving rational equations, rational equations, equations with rational expressions

Rational Equations, Level II.

Purpose: This is intended to refresh your skills in solving more complicated rational equations. If these are too difficult, try doing the DLA, Rational Equations, Level I, first.

Activity: Work through the following activity and examples. Do all of the practice problems before consulting with a tutor.

To solve equations containing rational expressions, we are going to use a property of equations: multiply both sides of the equation by any non-zero expression, and the solutions to the old equation are contained in the solutions to the new equation.

Example 1 $\frac{3}{x-5} = \frac{4}{x}$ The LCD is $x(x-5)$, and $x \neq 5$ or 0

$$x(x-5) \cdot \frac{3}{x-5} = x(x-5) \cdot \frac{4}{x}$$
$$x \cdot 3 = (x-5) \cdot 4$$
$$3x = 4x - 20$$
$$-x = -20$$
$$x = 20$$

Practice 1a $\frac{x}{x-2} = \frac{4}{3}$

Practice 1b $\frac{2}{x} = \frac{1}{x+1}$

Did you get 1a) 8 and 1b) -2 ? Very good!

Example 2 $\frac{x}{x+1} = \frac{x-1}{x-2}$ The LCD is $(x+1)(x-2)$, and $x \neq -1$ or 2

$$(x+1)(x-2) \cdot \frac{x}{x+1} = (x+1)(x-2) \cdot \frac{x-1}{x-2}$$

$$(x-2) \cdot x = (x+1)(x-1)$$

$$x^2 - 2x = x^2 - x + x - 1$$

$$-2x = -1$$

$$x = \frac{1}{2}$$

Practice 2a $\frac{x+2}{x+3} = \frac{x}{x-4}$

Practice 2b $\frac{6x-2}{2x-1} = \frac{9x}{3x+1}$

Did you get 2a) $-\frac{8}{5}$ and 2b) $\frac{2}{9}$?

Example 3 $\frac{x}{x+1} + \frac{1}{2x+1} = 1$ LCD is $(x+1)(2x+1)$, and $x \neq -1$ or $-\frac{1}{2}$

$$(x+1)(2x+1) \cdot \frac{x}{x+1} + (x+1)(2x+1) \cdot \frac{1}{2x+1} = (x+1)(2x+1) \cdot 1$$

$$(2x+1) \cdot x + (x+1) \cdot 1 = (x+1)(2x+1)$$

$$2x^2 + x + x + 1 = 2x^2 + x + 2x + 1$$

$$2x = 3x$$

$$-x = 0$$

$$x = 0$$

Practice 3 $\frac{2}{x-1} + \frac{x}{x+1} = 1$

Did you get $x = -3$?

Example 4 $\frac{x}{x+4} - \frac{x}{x-4} = \frac{x+36}{x^2-16}$

LCD is $(x+4)(x-4)$, and $x \neq -4$ or 4

$$(x+4)(x-4) \cdot \frac{x}{x+4} - (x+4)(x-4) \cdot \frac{x}{x-4} = (x+4)(x-4) \cdot \frac{x+36}{(x+4)(x-4)}$$

$$(x-4) \cdot x - (x+4) \cdot x = x+36$$

$$x^2 - 4x - x^2 - 4x = x+36$$

$$-8x = x+36$$

$$-9x = 36$$

$x = -4$, which makes the denominator zero. Since there are no other solutions, the answer is "No Solution."

Practice 4 $\frac{4}{x-3} + \frac{2x}{x^2-9} = \frac{1}{x+3}$

Hint: Factor your denominators first. Use this information to find the LCD and domain restrictions.

Did you get no solution also?

Example 5 $\frac{x+1}{x+5} - \frac{2x+1}{x-2} = \frac{5-x^2}{x^2+3x-10}$

LCD is $(x+5)(x-2)$, and $x \neq -5$ or 2

$$(x+5)(x-2) \cdot \frac{x+1}{x+5} - (x+5)(x-2) \cdot \frac{2x+1}{x-2} = (x+5)(x-2) \cdot \frac{5-x^2}{(x+5)(x-2)}$$

$$(x-2)(x+1) - (x+5)(2x+1) = 5-x^2$$

$$(x^2 - x - 2) - (2x^2 + 11x + 5) = 5 - x^2$$

$$x^2 - x - 2 - 2x^2 - 11x - 5 = 5 - x^2$$

$$-x^2 - 12x - 7 = 5 - x^2$$

$$-12x - 7 = 5$$

$$-12x = 12$$

$$x = -1$$

Practice 5 $\frac{x+3}{x-5} + \frac{2x^2+6}{x^2-7x+10} = \frac{3x}{x-2}$

Did you get $x = 0$? Now time to try some on your own.

Problems

1. $\frac{2}{x-5} = \frac{4}{x^2-25}$

2. $\frac{2}{x-1} + \frac{x}{x+1} = 1 - \frac{1}{x^2-1}$

3. $\frac{1}{x^2-3x} = \frac{2}{x^2-9}$

4. $\frac{2}{3x+6} = \frac{1}{6} - \frac{1}{2x+4}$

5. $\frac{x+2}{x^2+7x} = \frac{1}{x+3}$

6. $\frac{2}{x+3} = \frac{1}{x^2-4x-21} + \frac{3}{x-7}$

7. $\frac{4x+1}{x} - \frac{x}{x+1} = 3$

8. $\frac{10}{x^2-4x} - \frac{4}{x} = \frac{5}{x-4}$

Review: Meet with a tutor to verify your work on this worksheet and discuss some of the areas that were more challenging for you. If necessary, choose more problems from the homework to practice and discuss with the tutor.

For Tutor Use: Please check the appropriate statement:

_____ Student has completed worksheet but may need further assistance. Recommend a follow-up with the instructor.

_____ Student has mastered topic.