

## Introduction to Rational Expressions

### Definition of a Rational Expression

A rational expression is the quotient of two polynomials, such as shown in these three examples. (Remember that a monomial is a special type of polynomial.)

$$\frac{3x^2 - 27}{x^3 - 2x^2 + x}$$

$$\frac{15r^2s^4}{13rs^5}$$

$$\frac{3x^5 - x^3 + 9x - 5}{17x}$$

### Undefined Rational Expressions

A rational expression is undefined when values of the variable make the denominator equal to 0.

- 1) For each rational expression, find the value(s) of  $x$  that make each undefined. You may first have to factor as needed. If no values for  $x$  exist, then indicate with the word "none".

Example:  $\frac{5x}{2x-8}$

Solution:  $\frac{5x}{2(x-4)}$

$x=4$

a)  $\frac{7-x}{x(x+3)}$

b)  $\frac{8x}{x^2-49}$

c)  $\frac{x+5}{2x^2+5x-3}$

d)  $\frac{2x-1}{x^2+100}$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

### Reducing Simple Rational Expressions

When reducing rational expressions to lowest terms, remember the following three formats:

$$\frac{a+b}{b+a} = 1, \quad \frac{a-b}{-b+a} = 1, \quad \frac{a-b}{b-a} = -1 \quad (\text{for } a \neq b)$$

- 2) Use the formats above to circle the letter of any rational expression whose reduced form is equivalent to 1.

a)  $\frac{5-a}{5+a}$

b)  $\frac{8+y}{y+8}$

c)  $\frac{m-4}{4-m}$

d)  $\frac{-2+p}{-p+2}$

e)  $\frac{z-6}{-6+z}$

3) Use the formats of the previous page to circle the letter of any rational expression whose reduced form is equivalent to  $-1$ .

a)  $\frac{5-a}{5+a}$

b)  $\frac{-8+y}{y-8}$

c)  $\frac{m-4}{4-m}$

d)  $\frac{-2+p}{-p+2}$

e)  $\frac{z-6}{-6+z}$

### Reducing Rational Expressions

To help in reducing rational expressions, remember these 3 steps:

- 1) Factor the numerator
- 2) Factor the denominator
- 3) Look for factors of the numerator and denominator that can be reduced to 1 or  $-1$ , as done in the previous exercises.

4) Use the steps above to reduce each rational expression to lowest terms.

Example:  $\frac{18x - 9x^2}{3x - 6}$

Solution:  $\frac{9x(\cancel{2-x})}{3(\cancel{x-2})} = \frac{9x}{3} \cdot (-1)$   
 $= -\frac{9x}{3} = \boxed{-3x}$

a)  $\frac{9a^2 - 9b^2}{2a + 2b}$

b)  $\frac{y^2 + y - 12}{3y^2 + 12y}$

c)  $\frac{p^3 - 3p^2}{(3-p)^2}$

d)  $\frac{4 + 2d}{3d^2 - 18d + 24}$